Between Instrumentalism and Fundamentalism about Reformulations

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Draft of June 15, 2023

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1 Introduction

Throughout science, mathematics, and engineering, we often have multiple, compatible methods for solving problems. For each theory that scientists advance, they typically develop multiple ways of expressing or formulating its physical content. Often, the motivations for reformulating are practical: scientists wish to solve problems more quickly, simply, or elegantly. Sometimes, the aim is explicitly to clarify conceptual foundations, often by applying new mathematical techniques. Either way, the results of reformulating are a significant aspect of scientific progress. Reformulations often change how we understand the world, spawning new areas of research that probe the properties and scope of the reformulated theory. Similar remarks apply to reformulations in mathematics. These often lead to new proofs of old theorems, sometimes stemming from unexpected connections between seemingly disparate mathematical domains.

Insofar as we look to the sciences to motivate norms of inquiry, we should consider norms governing when to reformulate. Under what conditions is it wise to reformulate an existing problem-solving procedure or theory?[1] Schematically, utilitarianism provides a straightforward answer to this decision-theoretic question: whenever the expected utility of reformulating outweighs the expected utility of sticking with known problem-solving strategies, taking into account epistemic, practical, and moral factors. But this consequentialist answer is only a schema. Determining when it is wise to reformulate still requires assessing and weighing various values involved. Here, I focus on an axiological question rather than a decision-theoretic one: what is the nature of the value(s) that reformulations provide? Answering this axiological question is prior to determining when it is wise to reformulate.

One kind of reformulation provides an obvious sort of epistemic value. Some reformulations allow us to solve a problem that we couldn't solve before, providing knowledge that we could not have obtained otherwise. Theory change in science provides a paradigmatic example: quantum mechanics answers questions that classical mechanics simply does not resolve. Of two competing or rival formulations, one can provide answers that are closer to the truth. By positing rival ontologies, competing formulations straightforwardly lead to different ways of understanding the world. It is no surprise then that philosophy of science and the foundations of mathematics have historically focused on competing formulations of a given subject matter.

But what should we make of cases where two formulations are *compatible*, in the sense that we are not forced to choose between them? In these cases, we can simultaneously accept and use either formulation. They provide neither competing ontologies nor competing descriptions or predictions. Instead, they provide logically consistent problem-solving procedures for a shared class of problems. Physics supplies a wellspring of examples. Within classical mechanics, there are no less than five ways of formulating a large variety of problems, including the Newtonian, Hamiltonian, Lagrangian, Hamilton–Jacobi, and Routhian formulations of classical mechanics (Abraham and Marsden 1978; Arnold 1989). These formulations differ in their mathematical strategies for solving the equations of motion for classical systems, and—within their shared domain of applicability—they describe the same physical states of affairs. Similarly, non-relativistic quantum mechanics can be formulated in a variety of distinct mathematical garb, including wave mechanics, matrix mechanics, density operators, and path integrals

[1]For some norms of inquiry in connection to problem-solving, see Hookway (2007) and Friedman (2020).

(Styer et al. 2002). In what follows, my chief aim is to assess the value of compatible formulations and problem-solving procedures, abbreviating these simply as *reformulations*.

2 A Spectrum of Responses

The value of compatible reformulations is puzzling for at least the following reason. For a given problem, no particular formulation is necessary for providing a solution. Any compatible formulation would suffice. In this way, each compatible formulation seems to render the others dispensable for the purposes of problem-solving. Nevertheless, many reformulations seem to constitute a particular kind of intellectual progress, deepening our understanding.

To characterize the value of reformulating, I will consider a spectrum of philosophical positions. We can visualize these positions as lying along a continuum from maximally deflationary to maximally inflationary, i.e. involving substantial metaphysical commitments. My goal is to defend a position I call *conceptualism*, occupying a middle ground between these extremes.[2] Conceptualism illuminates a particular kind of non-practical, epistemic value that compatible formulations provide, what we might call *intellectual value*. This comprises aspects of what many would call the 'purely epistemic' (Sosa 2015, pp. 45, 172), although my aim is not to adjudicate the bounds of the epistemic.

The seemingly most deflationary position denies that compatible reformulations provide any kind of value beyond mere convenience. According to what I will call *conventionalism*, this is all there is to say about reformulations. Reformulations provide convenient footholds for forging ahead, facilitating the solution of problems we could solve with other methods if only we were willing to sacrifice the time and energy.[3] Conventionalism holds that there is nothing deep or epistemically significant about reformulations. They merely amount to a different choice of convention, no different in kind than a change in notation. Seen through this lens, the seeming intellectual triumphs of wholesale theoretical reformulations are simply one notational change after another, convenience piled atop convenience. Conventionalism holds that reformulations differ

[2]Conceptualism emphasizes the role that concepts play in theory reformulation and understanding, including what Kenneth Manders (2008, unpublished) calls "expressive means." These include the mathematical, linguistic, diagrammatic, and notational resources we use to express theories. I do not intend to endorse *conceptualism about universals*, despite some interesting analogies.

[3]A deflationary attitude is attractive for at least some notational choices, particularly what I'll describe as trivial notational variants. North describes—but does not fully endorse—this attitude in her discussion of coordinate systems: "Coordinate systems are labeling devices, tools that we impose...Since many such descriptive tools can be used, we tend to choose one for reasons of convenience" (North 2021, p. 22).

only in degree—rather than kind—from trivial notational changes.[4]

Conventionalism belongs to a family of views that I will call *instrumentalism*. What these views have in common is reducing the value of reformulations entirely to their instrumental value for accomplishing other epistemic or practical aims.[5] On the practical side, such aims include solving problems quickly, easily, or with fewer computational resources, along with manipulative control over target systems. On the epistemic side, such aims include prediction, discovery, descriptive adequacy, and increasing our credence or confidence in solutions to problems (such as via corroboration or by reducing the risk of error). I leave open whether some practical aims are epistemic as well. Goldman, for instance, classifies problem-solving speed or efficiency as epistemic (1986, p. 122). Regardless, I contend that we can isolate a non-practical dimension of the epistemic, namely *the intellectual*.

Instrumentalism contends that although reformulations are one method for achieving these practical and epistemic goods, reformulations are not constitutive of these goods. In this way, reformulations remain dispensable at least in principle. In Section 5, I argue that various versions of instrumentalism fail to respect a key aspect of scientific and mathematical practice, namely an intuitive distinction between trivial notational variants and non-trivial reformulations.

At the other extreme of our continuum lies *fundamentalism*. It proposes a metaphysical picture similar to David Lewis's. Lewis posits that some properties belong to an elite set of *perfectly natural properties*, with physics aiming to provide a partial inventory of these (1983, pp. 357, 364). Ted Sider speaks instead of a theory's conceptual structure, which must match the structure of reality in order for the theory to be "fully successful" (2011, p. vii). Sider's framework suggests that two formulations of a theory can state the same truths about the world while nonetheless disagreeing about which concepts are more fundamental, i.e. more joint-carving (2011, p. 5). According to Sider, successfully describing fundamental structure leads to greater epistemic value. These pictures motivate a metaphysically-committal account of the value of reformulations. Insofar as reformulating is sometimes constitutive of writing a theory in more joint-carving terms, fundamentalists can interpret some reformulations as non-instrumentally valuable.

For those willing to endorse additional metaphysical commitments, fundamentalism

[4]Some deflationary positions about the nature of scientific representation might be seen as inspiring or motivating conventionalism. See for instance Cohen and Callender (2006).

[5]See Korsgaard's (1983) distinction between instrumental value vs. final value (i.e. non-instrumental value, which need not be intrinsic). This kind of instrumentalism has been developed to give a deflationary account of the value of scientific explanations (van Fraassen 1980, pp. 93–4; Lombrozo 2011).

offers a non-instrumentalist account of the value of reformulating. However, it comes at the cost of difficult problems of epistemic access. As I argue in Section 6, these epistemic access problems partly spoil the positive story that fundamentalism can tell. The account I develop occupies a middle ground between instrumentalism and fundamentalism. Section 7 dubs this third strategy *conceptualism*: it focuses on how reformulations improve our epistemic position with regards to solving problems. I will argue that reformulations have non-instrumental value simply in virtue of how they restructure problem-solving. Successful reformulations clarify what we need to know to solve problems, improving our understanding of the world. Like instrumentalism, my account does not require substantial ontological commitments. Like fundamentalism, it accommodates the intuition that many reformulations are more than just instrumentally valuable.

A rival middle ground position—*explanationism*—holds that reformulations can be valuable in virtue of providing alternative explanations. Due to the vast number of different accounts of scientific explanation, explanationism provides a schema, to be filled in with a particular account of explanation. Different accounts of explanation give rise to different versions of explanationism. For this reason, it is logically difficult to argue decisively against explanationism. Nonetheless, I will consider a general problem that seemingly afflicts all versions of explanationism: reformulations manifest a number of differences that *prima facie* do not appear to be matters of explanation. Instead, these differences involve changes to the epistemic structure of problem-solving. They involve changes to how scientists and mathematicians go about structuring a search space through different inferential rules. Hence, I believe that a general account of reformulating requires focusing on how formulations structure problem-solving. Answering explanatory why-questions is, after all, just one kind of problem inquirers face.

Of course, nothing prevents fundamentalists or explanationists from adopting my conceptualist analysis but wanting to add more. A fundamentalist might wish to append additional commitments to fundamental structure. An explanationist might wish to append additional commitments to explanatory differences. My view is not incompatible with either of these augmentation strategies. Instead, conceptualism stands opposed to either fundamentalism or explanationism *being the end of the story* regarding the value of reformulations. My goal is to show that on their own, various versions of instrumentalism, fundamentalism, and explanationism provide inadequate accounts of reformulation. These negative arguments motivate a need for conceptualism as a positive account of the value of reformulations.

To make headway on the axiological question, Section 3 provides two simple illustrations of compatible formulations. Already in these cases, we see a range of values that compatible reformulations might manifest. Section 4 uses these examples to motivate three desiderata that any satisfying account of reformulations must satisfy. Subsequent sections argue that of the various accounts considered, only my preferred position—conceptualism—meets these three desiderata.

3 Two Simple Illustrations

Scientific reformulations are often rich and complex, involving advanced concepts from mathematics or sundry sciences. While inherently interesting, such examples require a wealth of background knowledge to assess. Fortunately, a couple simple examples illustrate characteristic features that arise.

Consider the following problem, discussed in the cognitive science literature on problem-solving and expertise (Goldman 1986, p. 132). Two trains—located at stations 50 miles apart—both head toward each other at 25 miles per hour. While they are moving, a bird flies back and forth between them at 100 miles per hour. The problem is to figure out how many miles the bird travels before the trains meet. One relatively *hard approach* to this problem involves calculating the distance the bird flies on each round-trip between the two trains. Stipulating that the bird always takes the shortest distance between the trains, one can determine the overall distance by summing a geometric series, with a term for each leg of the journey. An *easy approach* to solving this problem involves simply determining how long the bird is in flight. This equals the amount of time it takes for the trains to reach each other, namely, one hour. Hence, the easy approach entails immediately that the bird travels 100 miles as it flies between the trains.[6]

As a second example, consider an application of Gauss's law in electromagnetism. We are handed a ball containing static point charges of total charge Q. Our task is to quantify the strength of the electric field coming out of the ball. In other words, we need to determine the *electric flux* Φ_E, defined as the integral of the electric field $\mathbf{E}$ over the surface.[7] Naïvely, it would seem that to calculate the flux we need to know

[6]The *mutilated checkerboard problem* provides a similar example: after removing the squares from two opposite corners of a checkerboard, can the remaining squares be tiled with 31 dominoes? For discussion and additional examples, see Bilalić et al. (2019).

[7]More precisely, the electric flux Φ_E through a closed surface S is the surface integral of the component of the electric field normal to the surface, i.e. we integrate the scalar product of the electric field vector $\mathbf{E}$ with the differential of the normal vector to the surface d$\mathbf{a}$: $\Phi_E \equiv \oiint_S \mathbf{E} \cdot d\mathbf{a}$.

the electric field vector at each point passing through the surface. And to determine these electric field vectors, it would seem that we need to know the exact distribution of charges within the ball. Incredibly, Gauss's law shows us that we in fact do not need to know anything about either the charge distribution or the electric field to determine the flux. Instead, the electric flux simply equals the total amount of charge contained within our surface divided by a constant ε_0, known as the vacuum permittivity. Hence, knowledge of ε_0 and the total charge Q suffices for knowing the flux.[8]

In both cases, we have two compatible ways of solving the same problem. The procedures do not disagree about the way the world is. They provide the same answer to the problem and ultimately for the same physical reasons, albeit differently organized. Our axiological question is the following: what value is there to having more than one approach to solving the same problem? What do we gain by reformulating a problem-solving procedure or theory?

Instrumentalism contends that reformulating is not valuable for its own sake but merely as a means to other practical or epistemic ends. In each of our two illustrations, both formulations solve the same problem, so locally we do not have any non-practical epistemic differences. Each compatible formulation is as good as the other when it comes to obtaining the epistemic goods of (approximate) truth, prediction, or knowledge. The remaining differences between the formulations seem to be practical ones, such as differences in computational simplicity, efficiency, and convenience.

For instance, it is easier and faster to solve the bird–train problem by figuring out how long the bird is in flight than by calculating a geometric series. Likewise, it is easier and faster to apply Gauss's law to determine the electric flux than to painstakingly apply Coulomb's law. The easier methods may in turn decrease the risk of making a calculational mistake, but this is an epistemic difference in-practice, rather than in-principle. Later, I will consider whether global differences in problem-solving fruitfulness allow instrumentalism to draw epistemically significant differences between formulations. Perhaps one formulation generalizes to a wider range of phenomena, leading to increased instrumental value. For reasons considered in Section 4, I will argue that differences in fruitfulness still miss important epistemic differences between the approaches.

By contrast, on Lewis's fundamentalist framework, a formulation does better the

[8]For systems with appropriate symmetry, Gauss's law supplies another simple compatible reformulation. In such cases, we can calculate the electric field itself purely algebraically, eliminating the need for integration. In contrast, a non-symmetry-based approach would apply Coulomb's law and a superposition principle for electric fields, integrating for the electric field.

closer it comes to a canonical language that carves nature at its joints. A concept carves nature *perfectly* at its joints only if it is fundamental, but joint-carving is not an all or nothing affair. Instead, different concepts within the special sciences can be more or less joint-carving (Lewis 1983, p. 347). Sider enriches this picture by arguing that differences in joint-carving generate differences in the epistemic value of formulations. Given two languages for describing the world, if one of them carves nature better at the joints, then it has epistemic value that the other one lacks. Sider illustrates this in the context of the predicates green and grue, claiming that "it's *better* to think and speak in joint-carving terms. We ought not to speak the 'grue' language, nor think the thoughts expressed by its simple sentences" (2011, p. 61).

In the case of the bird and the trains, it is plausible that neither formulation is more joint-carving than the other. The geometric series approach keeps track of the causal details of the bird's trajectory, while the easy approach shows that we do not need this information to solve the problem. Yet, neither approach is obviously more fundamental. In cases like this, a fundamentalist might agree with an instrumentalist that this reformulation has no more than instrumental value.

The Gauss's law case is more interesting. As one of Maxwell's laws of electrodynamics, Gauss's law plausibly is more fundamental than Coulomb's law. Gauss's law is related to conservation principles, which themselves have a close connection with laws of nature and fundamental symmetries (Strocchi 2013, Ch. 7). Additionally, Gauss's law applies to not only static but also moving charges, and it is therefore more general than Coulomb's law. A fundamentalist might view this difference in fruitfulness as evidence that the Gauss's law approach gets closer to fundamental joints in nature.[9]

On the view I defend in Section 7, we can grant both that reformulations have instrumental value and even that they could—for all we know—have epistemic value coming from tracking fundamental structure. What matters is that we can be sure of one source of their non-instrumental epistemic value: reformulations clarify what we need to know to solve problems. By changing our epistemic situation, reformulations accrue epistemic value independently of any further metaphysical role they might play. In short, a significant reformulation leads to a different way of understanding the world. This is in contrast to trivial or insignificant reformulations, considered in the next section.

[9]As Tappenden (2008) notes, defenders of joint-carving may take differences in fruitfulness or fertility as evidence that one formulation is more fundamental than another (see Section 6). I remain neutral on whether fruitfulness plays this evidential role, at least when it comes to metaphysically robust notions of 'fundamental.' See also Nolan (1999), who argues that fertility is not a fundamental virtue.

4 Three Desiderata

I will argue that any satisfying account of reformulations must meet three desiderata. First, it must distinguish trivial notational variants from significant reformulations. Whereas some reformulations are merely matters of arbitrary, conventional choices, others appear to be epistemically significant. Second, a successful account must make sense of local differences between reformulations that arise when solving the same class of problems. Although reformulations often lead to differences in solving wider classes of problems, appealing only to these global differences does not address important local differences. Finally, the criteria that an account employs ought to be epistemically accessible. An account will be less satisfying insofar as it appeals to features of the world that might readily elude us. This section independently motivates these three desiderata. Sections 5 and 6 argue that both instrumentalism and fundamentalism fall short of meeting them.

Not all reformulations are epistemically significant. Some amount to nothing more than trivial notational variants. These include simple notational substitutions for typographical preference, the use of a right-handed rather than a left-handed coordinate system, conventions for summation, etc. I take it as a datum of scientific and mathematical practice that these trivial notational variants are epistemically insignificant. At the very least, they are much less epistemically valuable than paradigmatic cases of reformulation, including the two simple cases presented in Section 3. A successful account of compatible reformulations must provide principled grounds for distinguishing trivial notational variants from significant reformulations, affording greater epistemic value to the latter. This requirement supplies the *first desideratum*. To satisfy it, an account must avoid both (i) over-generating cases of significant reformulations (e.g. by classifying *all* reformulations as epistemically significant) and (ii) under-generating such cases (e.g. by classifying all reformulations as trivial notational variants).

To meet the first desideratum, an account must provide a principled distinction between clear cases of trivial vs. significant reformulations. This does not require providing necessary and sufficient conditions, since there may be vague cases that do not fall neatly into either category. Instead, it suffices to justify the datum that there is an epistemically significant difference between trivial vs. significant reformulations, with the latter being objectively more epistemically valuable (at least in clear cases). This distinction is objective in the sense that its truth does not depend on how agents feel or what they believe about it. Regarding the meaning of "epistemically significant" or "epistemic differences," there are many candidates, given the contested nature of the word 'epistemic' (Cohen 2016). Different accounts may specify different meanings for these terms. I describe my preferred account in Section 7, which focuses on non-practical dimensions of answering questions, i.e. solving problems.

The second desideratum constrains what we can appeal to when meeting the first. In clear cases, we can distinguish trivial from significant reformulations at the local level of individual problems or problem-types. This is another apparent datum of epistemic inquiry that any satisfying account must save. Given two compatible reformulations, there is a class of problems that they both solve. Within this shared domain of problems, significant reformulations display an epistemic difference, while trivial reformulations do not. Since these epistemic differences arise locally, we should account for them through local aspects of the formulations. It should not be necessary to consider global differences in fruitfulness or problem-solving scope. Unless shown otherwise, we should assume that these global differences arise from differences at the local level of solving individual problems. The second desideratum embodies these demands: a satisfying account of reformulations must provide local criteria for distinguishing trivial vs. non-trivial reformulations. Section 5 shows how the first two desiderata pose a serious problem for instrumentalism.

Besides the need to locally distinguish trivial from significant reformulations, a *third desideratum* presents itself: the criteria of significance should be epistemically accessible. To the extent that there are manifest epistemic differences between trivial and non-trivial reformulations, the criteria we use to explicate these differences should be manifest as well. Our account of reformulation should not be hostage to the lucky success of risky inferences. An account with epistemically inaccessible criteria may have the resources to address the first two desiderata, but it would be difficult to determine when the criteria are met. Accounts of reformulation that rely on risky inferences will face problems of underdetermination, leading to skeptical scenarios. The more difficult it is to determine whether the criteria are satisfied, the more severe these skeptical scenarios will be. In science, these worries about underdetermination are well-founded: there are numerous historical examples of scientists making needlessly risky inferences that were shown to be unfounded.[10] This is not an idle philosopher's skepticism. There are principled, practice-based reasons for seeking to avoid risky inferences whenever possible.

[10]Examples include Newton's inference from absolute acceleration to the existence of absolute velocity, 18th-century inferences to the existence of caloric as the carrier of heat, and 19th century inferences to the existence of an aether for the propagation of light as an electromagnetic wave.

An additional reason favors the third desideratum. Appraising compatible formulations is a challenge facing philosophers of many different temperaments, from constructive empiricists to those willing to posit Aristotelian essences. Ideally, an account of reformulation should have a widely-acceptable minimal core. This core should be as minimal in its ontological commitments or posits as possible. Nothing precludes those with additional metaphysical commitments from embellishing this account further, but it is harder to deconstruct a more metaphysically committal account into a version acceptable for the a-metaphysical. Section 6 shows how this third desideratum severely limits the appeal of fundamentalism, at least as the core of an account of reformulations.

5 Problems facing Instrumentalism

Recall that instrumentalism assesses reformulations based entirely on their instrumental value for various epistemic or practical aims. This suggests the following *instrumentalist criterion* for distinguishing trivial from non-trivial reformulations: a reformulation is significant if and only if it leads to an instrumentally valuable difference. Since these instrumentally valuable differences between formulations are epistemically accessible, instrumentalism easily satisfies the third desideratum. The challenge for instrumentalism is to satisfy the first desideratum without running afoul of the second. In other words, instrumentalism must distinguish between intuitive cases of trivial and non-trivial reformulations without either (i) over-generating cases of significant reformulations or (ii) appealing solely to global differences in problem-solving scope or fruitfulness. I will argue that the various instrumentally valuable differences each violate at least one of these conditions.

First, consider practical differences in convenience, such as problem-solving speed or ease of solution. Although significant reformulations often differ along these dimensions, so do paradigmatic cases of trivial notational variants. For instance, we find it extremely difficult to read mirror images of words.[11] Similarly, scientists sometimes develop strong psychological preferences for certain notational conventions. For instance, physicists working in particle physics phenomenology tend to use a different space-time metric convention than those working in general relativity or string theory. The former tend to use a mostly minus $(1, -1, -1, -1)$ metric while the latter use a mostly plus

[11]For an illustration, see Wittgenstein (2009 [1949], p. 209), remark 151 of *Philosophy of Psychology*. Framing effects from the presentation of statistics in terms of decimals or ratios provide a further example. See Kahneman (2011).

$(-1, 1, 1, 1)$ metric. Although just a choice of convention, "some physicists approach this issue with almost religious conviction" (Burgess and Moore 2006, p. 518). There are many compelling practical reasons to prefer one convention over the other, based on the kinds of problems that most commonly arise in either domain. An instrumentalism focused on these kinds of practical differences would diagnose these two metric conventions as significant reformulations. Such verdicts would vastly over-generate the class of significant reformulations, thereby running afoul of the first desideratum. Even if these practical differences between trivial notational variants are sometimes important, they still appear to be *different in kind* from the intellectual differences that conceptualism highlights.[12]

A structurally similar objection applies to versions of instrumentalism that focus on in-practice epistemic differences such as reducing the risk of error, or increasing one's degree of confidence in a solution. For instance, most people are less likely to make a calculational error using the easy approach to the bird–train problem than the geometric series approach. Yet, we also see reductions in the risk of error when using a trivial notational variant that we are more comfortable or familiar with. Hence, this criterion does not distinguish trivial from significant reformulations. Similarly, we gain increased confidence in a solution whenever we solve a problem anew, whether using a trivial or a significant reformulation. This is no different than how double checking an answer can increase our confidence in it.[13]

Turning to differences in prediction, control, or descriptive adequacy, we see that these differences do not arise locally. By definition, two compatible formulations both solve a shared class of problems. Hence, they locally provide the same predictions, are equally approximately true, and provide the same degree of manipulative control. It is therefore difficult to see how there could be local differences along these dimensions.

To meet the first desideratum, instrumentalism seemingly must appeal to global differences, such as differences in fruitfulness. When we broaden our scope to consider how reformulations differentially generalize in different contexts, sometimes certain formulations succeed where others fail. For instance, the easy solution to the bird–train problem applies even to a bird executing exquisite loop-de-loops between the trains. In contrast, the geometric series solution requires that the bird fly in straight lines (other-

[12]Moreover, what counts as computationally simpler or more convenient is often a matter of taste or pedagogical training. Ideally, we would satisfy the first desideratum by giving an objective distinction between clear cases of trivial and significant reformulations.

[13]As Davidson notes, "it is often worthwhile to increase our confidence in our beliefs, by collecting further evidence or checking our calculations" (2005, pp. 6–7).

wise, we would require further information about the bird's trajectory). Similarly, the Gauss's law approach applies to moving charges, while the Coulomb's law approach requires that the charges are static. In each case, one formulation is more fruitful than the other, applying to a strictly wider range of problems.

No doubt, differences in fruitfulness are instrumentally valuable. They constitute differences in the predictions we can make and the phenomena we can save. However, they are not differences that arise at the local level of shared problem-solving. We should expect that these global differences are symptoms of underlying local differences in problem-solving. Appealing solely to global differences relinquishes the goal of identifying local differences that are *prima facie* significant. It would be more satisfying if we could accommodate global differences in terms of local, epistemically significant differences between formulations. We should give up on the second desideratum only if other promising accounts fail to meet it as well. For this reason, instrumentalism is not enough to account for the significance of reformulations. Instrumental differences are part of a larger story, but they are not the whole story.

6 Problems facing Fundamentalism

According to many scientific realists, science aims at the truth. Fundamentalism proposes a further aim for empirical inquiry: an ideal scientific theory must describe the world in a fundamental language.[14] Two descriptions of a subject matter can both be true, while one of them is more fundamental. Lewis contends that physics aims at providing an inventory of natural properties (1983, p. 357). According to Lewis, "the businesss [sic] of physics is not just to discover laws and causal explanations. In putting forward as comprehensive theories that recognize only a limited range of natural properties, physics proposes inventories of the natural properties instantiated in our world" (1983, p. 364). Likewise, Sider argues that describing the world in joint-carving terms leads to greater epistemic value than merely having a true theory:[15]

[14]Similarly for ideal logical and mathematical theories, leaving open whether these are about the world.

[15]Dasgupta (2018) has challenged Sider's contention that more fundamental descriptions necessarily have greater epistemic value. He argues that fundamentalists must *explain* where this epistemic value comes from, but that no explanation is forthcoming. However, I worry that Dasgupta's argument is self-undermining. Fundamentalists fail to meet Dasgupta's demand for an explanation only if we presuppose that a realist conception of explanation is desirable. This amounts to presupposing a value claim little different than what the fundamentalist is accused of presupposing. Hence, Dasgupta's own argument is subject to either circularity or an infinite regress. The fundamentalist can simply demand that Dasgupta explain why the fundamenalist owes an explanation of the epistemic value of approximating fundamental

> The goal of inquiry is not merely to believe truly (or to know). Achieving the goal of inquiry requires that one's belief state reflect the world, which in addition to lack of error requires one to think of the world *in its terms*, to carve the world at its joints. Wielders of non-joint-carving concepts are worse inquirers. (2011, p. 61)

Although neither Lewis nor Sider are explicitly concerned with compatible formulations, their commitments to fundamental structure suggest a *fundamentalist criterion* for assessing reformulations: a reformulation is epistemically valuable whenever it leads to a more joint-carving formulation. Using this criterion, fundamentalism straightforwardly meets the first two desiderata from Section 4. It proposes an objective epistemic difference between trivial notational variants and significant reformulations. Whereas trivial notational variants are equally joint-carving, significant reformulations exhibit a difference in fundamentality: namely, one formulation is more joint-carving than the other. Furthermore, the fundamentalist criterion of significance is local: these differences in fundamentality arise at the level of individual problem-solving. Fundamentalism thereby satisfies the second desideratum as well. Although evidence for differences in joint-carving might come from global considerations of fruitfulness, the differences themselves arise locally (if they arise at all).

The main problems facing fundamentalism arise from its substantial ontological commitments. Many empiricists and scientific anti-realists (and even some realists) disavow commitments to perfectly natural properties and fundamental structure. Relying on these commitments precludes fundamentalism from providing a minimal account of the value of reformulations. In response, a fundamentalist might be inclined to say: so much the worse for the metaphysically-averse. But there are independently compelling reasons to be wary of appeals to fundamental structure. One reason comes from fundamentalism itself: Occam's razor. If we can provide a positive account of reformulations with fewer metaphysical commitments, then this account will be simpler. Fundamentalists would then, by their own lights, have reasons to take such an account seriously. This provides one reason in favor of the conceptualist account I provide in Section 7.

More substantial metaphysical commitments typically involve posits that are less epistemically accessible. It is difficult to know if and when theory formulations track perfectly natural properties. Beyond appeals to intuition, fundamentalists can rely on theoretical virtues as evidence for greater fundamentality. Whether and when differences in virtues—such as simplicity or fruitfulness—track fundamentality seems ulti-

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structure. By his own lights, Dasgupta will not be able to meet this demand, so his own demand for explanation is self-undermining.

mately decided by appeals to philosophical intuition. Some scientific realists and fundamentalists may be sufficiently optimistic about these aspects of philosophical methodology. For them, these epistemic access problems may not be substantially more troubling than our access to physical unobservables posited by ordinary scientific theories. Nevertheless, an account of reformulations would be epistemically more secure if it did not rely on these controversial methodological commitments.[16] Ideally, we should seek an ontologically minimal account of reformulations that even empiricists can adopt. More metaphysically-committed philosophers then remain at liberty to invoke additional ontological commitments when assessing reformulations.

Epistemic access problems also lead to problems of underdetermination. Consider an epistemically possible world where neither the Gauss's law nor the Coulomb's law formulation is more fundamental than the other (see Section 3). This world is empirically indistinguishable from the one fundamentalists might take us to be in, where the Gauss's law formulation is putatively more fundamental. In either world, our physical theories make exactly the same predictions about both observables and unobservables. Yet, the two worlds disagree about whether the Gauss's law formulation is more fundamental, and hence about whether the two formulations are trivial notational variants. In the former world, the fundamentalist criterion of significance classifies the two formulations as trivial notational variants (at least *qua* fundamental structure). In the other, this criterion says that the formulations are significantly different. But since both worlds would be empirically indistinguishable, it is difficult to know which one we are in.[17]

As a result, fundamentalism makes the significance of compatible formulations hostage to empirically inaccessible facts about fundamental structure. Even worse, these inaccessible facts do not have any bearing on how the formulations appear to us. Metaphysical facts about differences in joint-carving do not change how we solve problems or understand the world using our theories. Fundamentalism meets the first two desiderata *in principle.* But in virtue of failing the third desideratum, fundamentalism makes it difficult to know when significant differences arise. To avoid underdetermination problems, we should strive for an account of compatible formulations that does not depend on relatively inaccessible facts, e.g. about fundamental structure. Even if we are mis-

[16]Cohen and Callender (2009, p. 13) argue that perfectly natural properties face additional problems of epistemic access beyond the usual skeptical challenges to knowledge of physical unobservables. See Woodward (2016, p. 1056) for additional criticisms against flatly invoking natural properties.

[17]van Fraassen (1975) develops a similar underdetermination problem for mathematical Platonism through his parable of the lands of Oz vs. Id. Cohen and Callender (2009) provide this kind of argument against the epistemic accessibility of perfectly natural properties.

taken or woefully ignorant about the world's fundamental structure, we should be able to satisfactorily interpret significant epistemic differences between reformulations.

7 Conceptualism

With myriad problems facing instrumentalism and fundamentalism, I now develop an account of reformulations that avoids these problems. Because it focuses primarily on how different concepts restructure problem-solving, I call my account *conceptualism*. Conceptualism accounts for the significance of reformulations in terms of how they structure problem-solving, based on the inference rules deployed. Section 7.1 provides examples of these inferential differences. They constitute differences in the inferential structure of a problem-solving procedure. Section 7.2 shows how conceptualism easily satisfies all three desiderata, providing a positive, local, and ontologically minimal account of reformulations. Finally, Section 7.3 considers in more detail the notion of sameness or equivalence of inferential structure.

7.1 Inferential structure

Consider the toy example from Section 3 involving a bird flying between two trains. The hard procedure requires knowing the distance the bird travels on each leg between the trains (or alternatively, the time spent on each leg). The easy procedure shows that we do not need to know the bird's detailed trajectory: it suffices to know the speed at which the bird flies and the amount of time spent flying. Similar considerations apply to calculating the electric flux emanating from a charged body using Gauss's law. This law shows that we do not need to know the distribution of charges within the charged object or the electric field at each point on the surface. Instead, it suffices to know the total amount of charge the object contains.

More complicated cases of reformulation also display differences in what we need to know to solve problems. In the quantum mechanics of atoms and molecules, we can often use symmetry arguments to solve problems without needing to know many details about a system's dynamics. In contrast, elementary methods that eschew appeals to symmetries require more detailed information to solve the same problems. The Lagrangian formulation of classical mechanics illustrates a similar moral. It tells us that to calculate the equations of motion for a classical system, we do not need to know the constraint forces acting on the system. In contrast, the Newtonian formulation requires knowledge of these constraint forces. My conceptualist account focuses on differences in what we need to know to solve problems, arguing that such differences in inferential structure underpin significant reformulations.

Each compatible formulation constitutes a plan for problem-solving, comprising a set of inferential steps. Each inference step takes us from an input set to an output, ultimately resulting in a solution. Two problem-solving plans are *inferentially equivalent* just in case what you need to know to carry out one plan is the same as what you need to know to carry out the other plan. For instance, what an English speaker needs to know to carry out the easy approach to the bird–train problem is the same as what a French speaker needs to know, even though the English speaker uses English and the French speaker uses French. The propositions that these two speakers need to know are the same, even though they are voiced in different languages, using different sentences. Similarly, two Turing machines are inferentially equivalent provided that they carry out an identical algorithm, even if their individual command lines are written in a different order or using different symbols. Section 7.3 further defends these equivalence claims.

To speak more precisely about inferential structure, it is convenient to introduce a three-part relation between (i) input information, (ii) an inference rule, and (iii) output information. Call these *inferential relations*. An inferential relation is satisfied provided that applying the inference rule to the input yields the output. Each inferential relation fixes what one needs to know to apply it, namely (i) the input information and (ii) the inference rule.[18] Two problem-solving plans are inferentially equivalent if and only if they have the same inferential relations. Hence, a sufficient condition for being inferentially inequivalent is having different inferential relations. This amounts to a difference in what one needs to know to solve problems using the two formulations.

Compatible formulations involving Arabic vs. Roman numerals provide a simple illustration of inferentially inequivalent plans. Imagine a lecture hall with 21 rows of 16 seats each. Our task is to determine how many people it can seat. Arabic numerals allow us to multiply 16 by 21 using a standard algorithm from grade school. This algorithm takes advantage of Arabic numeral's positional notation to modularize the problem into a series of single-digit multiplication and addition sub-problems, such as calculating six times two. To use this multiplication algorithm, one needs to know (i) 100 single-digit multiplication facts (i.e. the times table up to 9) and (ii) how to add Arabic numerals.

[18] I do not intend to take a stance here on whether knowledge-how reduces to knowledge-that. If one denies the reduction, then they can read "knowledge of the inference rule" as meaning "knowing how to use the inference rule."

Now, imagine reformulating this multiplication problem using Roman numerals, i.e. calculating XVI times XXI. Since Roman numerals are a sign-value system rather than a positional one, our familiar algorithm does not work.[19] We must rely instead on an inferentially inequivalent problem-solving plan. Rather than an addition table, we instead use seven simplification rules such as replacing "IIIII" by "V". We also use a multiplication table of 49 separate multiplication facts (such as L times L equals MMD), which must be augmented for factors above one million.[20] When it comes to figuring out that 16 times 21 equals 336, these two formulations display different inferential structures, characterized by differences in what one needs to know to solve the problem. They thereby amount to inferentially different plans for problem-solving.

In contrast, merely interchanging the symbol '5' everywhere with the symbol 'V' does not result in an inferentially different plan. With this symbol substitution, we could use either problem-solving plan described above. Hence, inferential (in)equivalence concerns not only the notation we use but also *how* we use that notation, i.e. the problem-solving plans supported by that notation or other relevant concepts.

7.2 Satisfying the three desiderata

Conceptualism proposes a straightforward criterion for assessing the significance of reformulations: a reformulation is significant (i.e. non-trivial) when it results in an inferentially inequivalent problem-solving plan. As argued in the preceding section, a sufficient condition for inferential inequivalence is that the two formulations differ in what one needs to know to apply them. I now show that this criterion satisfies all three desiderata from Section 4. It provides a principled distinction between intuitive cases of trivial and non-trivial reformulations that is both local and epistemically accessible.

I leave open whether meeting this criterion is also *necessary* for a reformulation to be significant in the relevant sense, i.e. setting aside practical dimensions such as problem-solving speed or reducing the practical risk of error. Conceivably, there might be two formulations that differ at the level of joint-carving but not at the level of what one needs to know to solve problems. If joint-carving differences do have objective epistemic value, then this would provide a separate sufficient condition for significance. However,

[19] In an additive system, the string represents the sum of its individual numerals. For simplicity, I do not consider subtractive notation such as "IV" for four, representing this instead as "IIII." Everything I say below could be adapted to this case. See Detlefsen et al. (1976).

[20] See Schlimm and Neth (2008, p. 2100) for details of this algorithm, which relies on the distributive law. Figuring out which of two numbers is greater also involves different inferential relations in these two formalisms; see Colyvan (2012, pp. 133–4).

as argued in Section 6, this metaphysical criterion is not epistemically accessible.

The first desideratum demands a principled distinction between trivial notational variants and significant reformulations. Unlike significant reformulations, two problem-solving plans that are trivial notational variants fail the above criterion: they have identical inferential structure, deploying the same inferential relations for solving problems.[21] Symbol substitution provides the simplest case: substituting every instance of a symbol α with a previously unused, arbitrary symbol β does not alter a formulation's inferential relations. Likewise, even though many scientists prefer to work in a right-handed coordinate system, working in a left-handed coordinate system preserves the same inferences. In relativistic theories, the arbitrary choice between a mostly positive or a mostly minus metric convention does not lead to inferential differences. Hence, these two conventional choices are trivial notational variants. This is the case despite the fact that many physicists have a personal—and sometimes subfield-wide—preference for one convention over the other. After discussing the other two desiderata, I return in Section 7.3 to the question of whether there is any sense in which trivial notational variants exhibit different inferential relations.

The conceptualist criterion also satisfies the second desideratum, which demands that we *locally* distinguish trivial from significant reformulations. Inferential differences arise at the local level of solving individual problems. We can assess whether two compatible formulations are inferentially distinct by considering their shared class of problems. We need not appeal to differences in their fruitfulness or scope. Differences in fruitfulness are no doubt also epistemically significant, but conceptualism shows how they arise from local differences in inferential relations. It is in virtue of restructuring our solution procedures that some formulations become more fruitful than others for certain classes of problems. Differences in fruitfulness are not a reason for significance; they are a symptom.

Finally, conceptualism satisfies the third desideratum by proposing a criterion that is epistemically accessible. Differences in inferential relations are not empirically underdetermined. We learn about them simply by analyzing how various formulations support problem-solving. For instance, when we discover a new way to solve a problem, we learn that we didn't need to know certain facts that an earlier formulation required.

In virtue of our relatively easy access to inferential structure, conceptualism avoids the underdetermination problems that afflict fundamentalism. Even in a world where

[21]Framed in terms of syntactic symmetries, the inferential structures/plans provided by two trivial notational variants are invariant under the reformulation.

we are radically wrong about which formulation is more fundamental, we will be right about many inferential differences. In contrast, fundamentalism relies on differences in fruitfulness or other super-empirical virtues as *evidence* for metaphysical differences. This involves making an inductively risky *inference* to the existence of underlying differences in fundamental structure. Conceptualism provides a method for appraising reformulations that avoids these risky inferences. Even anti-realists about physical unobservables can recognize inferential differences between formulations.

Scientific realists, fundamentalists, and others may hanker for a metaphysically deeper account of inferential structure. They may seek to ground inferential differences in explanatory differences or differences in fundamental structure. For instance, perhaps some inferential differences correspond to differences in what information is explanatorily relevant to the problem-solution. According to this explanationist proposal, information that *we do not need* to solve a problem is explanatorily irrelevant. In the case of the bird and the trains, we do not need to know the detailed trajectory that the bird takes. Some may therefore be inclined to say that such details are explanatorily irrelevant. However, one difficulty with this inference is that it takes us from considering the inferential structure of a formulation to considering more contentious explanatory relations in the world. Philosophers who support causal-mechanical accounts of explanation may have a different intuition. From a causal-mechanical standpoint, the bird's detailed trajectory explains the distance it travels. It remains explanatorily relevant, despite the fact that we do not need to know it in order to solve certain problems. Conceptualism shows that we can positively assess reformulations without resolving these kinds of explanatory disputes. Section 8 considers explanationism in more detail.

Nothing prevents philosophers with a more optimistic view of theoretical virtues from making further inferences about physical or metaphysical facts that ground inferential structure. They are welcome to do so if so inclined. Nevertheless, these additional commitments preclude fundamentalism and some forms of explanationism from providing a metaphysically minimal account of reformulations, based on epistemically accessible resources. If instrumentalism could meet the first two desiderata, it would already provide a minimal account. But as it stands, instrumentalism is inadequate. At the other extreme, fundamentalism commits us to more than necessary. Conceptualism, I have argued, is just right.

7.3 Sameness of inferential structure

My argument in Section 7.2 assumes that paradigmatic cases of trivial notational variants have the same inferential structure, i.e. they are *inferentially equivalent.* To characterize inferential equivalence, one must accommodate the following difficulty: even obvious cases of trivial notational variants require knowing slightly different things—in some sense of "different"—simply because they involve different notation. For instance, to solve a problem using a left-handed coordinate system, one needs to understand the relevant convention. This left-handed convention is *ipso facto* different than that of a right-handed convention. To take a linguistic analogy, knowing what "dogs bark" means requires knowing some English, while knowing what the synonymous expression "die Hunde bellen" means requires knowing some German.

Understanding any sentence requires understanding a notation. Yet the notation does not thereby become part of the content of the sentence. Whatever inferential differences exist between trivial notational variants, they wholly concern the notation rather than what we use the notation to represent. These reflections motivate an *aboutness criterion* for inferential equivalence: two formulations are inferentially equivalent just in case any differences in what an agent needs to know to apply the formulations are merely *about the notation*, rather than about the content of the formulations. Notation is a vehicle for communicating content, not the content itself.[22]

Rather than in terms of aboutness, we can characterize inferential equivalence using an account of synonymy of meaning. Different accounts of meaning may yield different accounts of inferential equivalence. For my purposes, it suffices to illustrate one such account, showing that it is possible to provide a principled distinction between trivial notational variants and significant reformulations. Since my account relies on an equivalence between problem-solving plans, it is natural to use Gibbard's (2012) account of meaning, which is based on a similar notion of planning.

Following Gibbard, we can understand the synonymy of "dogs bark" and "die Hunde bellen" as follows. Both sentences voice the same thought, which we can denote either as DOGS BARK or DIE HUNDE BELLEN. To believe the sentences are synonymous (in a given situation) is simply to plan to use "dogs bark" if I am an English speaker in those situations that I would plan to use "die Hunde bellen" if I were a German speaker, and vice versa. Hence, synonymy of meaning amounts to equivalence of plans. Although

[22]See North (2021, pp. 20, 32) for a related discussion of conventions or arbitrary choices in terms of aboutness. To make this criterion more precise, one could apply Yablo's (2014) account of aboutness.

the English and German speaker know different languages, once we abstract away these linguistic differences, they know the same thing, namely the thought that DOGS BARK.

Applying Gibbard's account of synonymy to inferential structure supports the conceptualist criterion I have been defending. Consider a problem that one can solve using either a left-handed or a right-handed coordinate convention. In the left-handed case, I appeal to an inferential relation (IR) expressed in the left-handed convention, denoted 'IR_{left}.' In the right-handed case, I appeal to an IR expressed in the right-handed convention, denoted 'IR_{right}.' IR_{left} and IR_{right} voice the same inferential relation provided that when working in a left-handed convention, I plan to use IR_{left} in the same situations as I would plan to use IR_{right} if I were working in a right-handed convention. Hence, although I technically need to know something different to work with IR_{left} rather than with IR_{right} (and vice versa), these two inferential relations are synonymous. The difference in what I need to know is wholly about my notation, rather than a genuine inferential difference in problem-solving plan.

Unlike trivial notational variants, significant reformulations provide different plans for solving problems. Ultimately, this is borne out as a difference in the inferential relations that they exploit or make available. For example, in the bird–train problem, someone using the hard formulation needs to determine the distance the bird travels on its first segment, second segment, etc. (or, alternatively, the time spent on each segment). They then need to know how to sum the distance on these segments, relying on an inference rule for summing an infinite geometric series. An agent following the easy formulation does not need to determine this information, nor rely on this inference rule. This is a genuine difference in the inferential structure of these problem-solving plans.

8 Problems with Explanationism

I have argued that conceptualism meets the three desiderata laid out in Section 4. It provides a middle ground between instrumentalism and fundamentalism about reformulations. Of course, other intermediate positions might meet these three desiderata as well. *Prima facie*, one approach that seems attractive involves tracking putative differences in explanation. Perhaps two compatible formulations are significantly different provided that they exhibit explanatory differences. I will call this schematic proposal *explanationism*. It satisfies the first desideratum by holding that trivial notational variants are explanatorily on a par, whereas significant reformulations manifest explanatory differences. Provided that these explanatory differences are local and epistemically accessible, explanationism will meet the second and third desiderata as well. In this section, I argue that conceptualism has important advantages over explanationism. In particular, conceptualism characterizes the epistemic differences between reformulations without taking a stand on the contentious topic of explanation.

Whether or not two compatible formulations have an explanatory difference depends on the nature of explanation. Different accounts of explanation give diametrically opposed verdicts on the simple examples that we have considered. Hempel's (1965) deductive–nomological account characterizes both the easy and hard approaches to the bird–train problem as equally explanatory: both appeal to the same law-like statement (distance as a function of rate and time), the same initial conditions, and provide equally rigorous derivations of the explanandum. Hence, a Hempelian explanationist would have to view these as trivial notational variants. On a causal–mechanical account of explanation, the hard approach to the bird–train problem might be viewed as more explanatory, since it explicitly tracks additional causal details. Likewise for the Coulomb's law approach to calculating electric flux, since this approach explicitly calculates the electric field from each individual charge. A unificationist account of explanation suggests the opposite verdict: by eliminating reference to these additional causal details, the simple approach to the bird–train problem and the Gauss's law approach both apply to a wider range of phenomena.[23] Despite combining aspects of causal and unificationist approaches, Strevens' (2008, p. 97) kairetic account of explanation would agree with the unificationist verdict here. Roughly, the kairetic account characterizes information that can be abstracted from a causal model—while still saving the phenomena—as being explanatorily irrelevant.

These disagreements illustrate the important role that philosophical assumptions play in assessing what information counts as explanatorily relevant. In contrast, we can recognize that one formulation does not require information that another requires, without presupposing further philosophical claims about explanation. Hence, we can more securely discern that reformulations display inferential differences than explanatory differences. Moving from a recognition of these inferential differences to claims about explanatory relevance requires further philosophical principles.

For instance, when we find out that knowledge of the distance traveled on each leg of the bird's journey is unnecessary for solving the bird–train problem, we might

[23]For this traditional dialectic between causal-mechanical vs. unificationist account of explanation, see Salmon (1998).

be tempted to infer that this information is explanatorily irrelevant. Doing so requires endorsing a philosophical principle like the following: contextually-unnecessary but causally efficacious information is explanatorily irrelevant. Proponents of causal–mechanical pictures of explanation may have different philosophical intuitions about whether this contextually-unnecessary information is explanatorily irrelevant. They might instead argue that tracking this information provides a deeper explanation, even if this deeper explanation is unnecessary for many purposes. My point here is a simple one: settling this sort of philosophical dispute is downstream from characterizing central epistemic differences between compatible formulations. We can account for many of the epistemic and methodological advantages of reformulations without settling these further questions about explanation or explanatory relevance.

Most accounts of explanation agree on at least one thing: explanations provide answers to why-questions.[24] Explanatory information describes the reasons why an event occurred or a fact is true. This aspect of explanation provides a second argument for viewing explanatory differences as logically downstream from the epistemic differences that concern conceptualism. Logically, why-questions form a proper subset of a larger category of scientific and mathematical questions. Not all problems take the form of requests for explanatory information or reasons why. Hence, not all problem-solving procedures provide explanations, even if they succeed at providing solutions. Questions about whether a solution procedure is explanatory typically go beyond whether it provides the correct solution. We see this, for instance, in the case of mathematics: a rigorous proof of a mathematical theorem may not count as explanatory. For instance, Lange (2009) argues that proofs by mathematical induction often fail to be explanatory.

In privileging conceptualism over explanationism, I do not deny that philosophical questions about explanation and explanatory relevance are important. My point is merely that various versions of explanationism could agree with my conceptualist analysis of reformulations, while disagreeing about the nature of explanation. Conceptualism is thereby better suited to provide a minimal core for an account of reformulations. Having adopted this minimal core, one can then defend further philosophical principles about explanation and explanatory relevance. In this way, my complaint against explanationism is similar to my complaint against fundamentalism: to assess important epistemic differences between reformulations, explanationism has to presuppose more than necessary.

[24]This includes both irrealist frameworks such as van Fraassen's (1980) pragmatic account and realist approaches such as Skow's (2016) causal–grounding account of reasons why.

9 Conclusion

Conceptualism holds that the value of reformulating comes from clarifying what we need to know to solve problems. Significant reformulations provide an inferentially different way of solving the same problem. This is in contrast to trivial notational variants, which do not alter what we need to know to solve problems. I have shown how conceptualism provides a middle ground between instrumentalism and fundamentalism, preserving the positive features of these accounts while avoiding their drawbacks.

Cast in terms of an aim of inquiry, conceptualism holds that inquirers ideally ought to clarify what they need to know to solve the problems that interest them. Doing so provides a kind of non-practical epistemic value, which we might call *intellectual value*. Intellectual value is importantly different from the kinds of practical value that can arise from good notational choices. For instance, I might have a strong preference to work in a right-handed coordinate convention, or to read ordinary rather than mirror-image text. I might be considerably faster or more reliable with one notation than the other. Such practical differences might strike some as being genuinely epistemic, an issue that I have left open here. Regardless, they are importantly different from the kinds of non-practical epistemic differences that significant reformulations provide. Such differences in problem-solving plans exist independently of anyone's preferences, comfort-level, speed, or risk of error. They exist even for ideal computers.

Of course, if one computer yields a solution in five minutes while another takes two weeks, that is practically important for belief-formation. However, if the two computers implement the same problem-solving plan (e.g. program), then there is no intellectually significant difference between them. Conceptualism neatly captures this distinction between practical and non-practical dimensions of epistemic value. By figuring out what we need to know to solve problems, we enhance our understanding of the world. The intellectual value of reformulating thereby stands independent of any downstream benefits such as greater fruitfulness, better explanations, or more fundamental descriptions.

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